Basic Engineering Mathematics

Determinant

Q1. Prove that $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

Q2. Prove that $\begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix} = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (a-b)(b-c)(c-a)$

Q3. Prove that $\begin{bmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{bmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

Q4. Solve the system of equation by using Cramer's Rule:

$x - y + 2z = -4, 3x + y - 4z = -6, 2x + 3y - 4z = 4$

Matrix

Q1. Given A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{bmatrix}$ Calculate AB and BA and show that AB ≠ BA.

Q2. $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$; Find $A^2 + 2A + 7I$

Q3. $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ $Find\ A^{-1}$

Q4. Solve the following system of equations by matrix method:

$x + y + z = 6, 2x - y + z = 3, x - 2y + 3z = 6$

Q5. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ $Find\ A^{-1}$

Vector

Q1. Find the unit vector in the direction of P(1,2,3) towards Q(4,5,6).

Q2. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$, then find $|\vec{x}|$

Q3. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}.\vec{b}=\sqrt{6}$

Q4. If $\vec{a}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$ and $\vec{b}=3\hat{\imath}-\hat{\jmath}+2\hat{k}$, show that $(\vec{a}+\vec{b})$ is perpendicular to $(\vec{a}-\vec{b})$

Q5. Show that the points A(1,2,7) B(2,6,3) and C(3,10,-1) are collinear.

Q6. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then find the value of

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$$

Q7. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Q8. Prove that the points A, B, and C with position vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}-5\hat{k}$, and $3\hat{\imath}-4\hat{\jmath}+4\hat{k}$ Form a right-angled triangle.

Q9. Find the area of triangle with vertices (1,1,2), (2,3,5) and (1,5,5).

Q10. Three forces $2\hat{\imath}+7\hat{\jmath}, 2\hat{\imath}+15\hat{\jmath}+6\hat{k}, 15\hat{k}$ acting on the point P with position vector $4\hat{\imath}-3\hat{\jmath}+2\hat{k}$ and displaces it to point Q with position vector $6\hat{\imath}+\hat{\jmath}-3\hat{k}$. Find the work done.

Q11. Two forces $\hat{\imath}+\hat{\jmath}-\hat{k}$ and $-2\hat{\imath}-\hat{\jmath}+\hat{k}$ are applied at the point $3\hat{\imath}-\hat{\jmath}$. Find the moment of the force system about the point $2\hat{\imath}+\hat{\jmath}-2\hat{k}$

Differential calculus

Q1. Evaluate (i) $\lim_{x\to a}\frac{x\sqrt{x}-a\sqrt{a}}{x-a}$ (ii) $\lim_{x\to a}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a^2+ax}}$

Q2. Evaluate $\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

Q3. Evaluate $\lim_{h\to 0}\frac{(a+h)^2 sin(a+h)-a^2 sina}{h}$

Q4. Show that $\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2}-\sqrt{1+sinx}}{\sqrt{2}cos^2 x}=\frac{1}{8}$

Q5. Evaluate $\lim_{x\to\frac{\pi}{2}}(secx-tanx)$

Q6. Find the derivative of *cosx* using first principle.

Q7. Prove that $f(x)=\begin{cases}x^2, \text{if } x\neq 1\\ 2, \text{ if } x=1\end{cases}$ is continuous at $x=1$

Q8. Find the derivative of $\sin^{-1}x$ using first principle.

Q9. Differentiate $\frac{4x^2-1}{(5-2x)^3}$ find the value of derivative at $x=2$

Q10. Differentiate the following functions $\frac{x}{3+\tan x}$

Q11. Find $\frac{dy}{dx}$ of $y=\frac{x\tan x}{\sec x+\tan x}$

Q12. Find $\frac{dy}{dx}$ of $y=e^{\tan^{-1}\sqrt{x}}$

Q13. Differentiate $\sin e^{x^3}$ w.r.t x using chain rule.

Q14. Find $\frac{dy}{dx}$ of $y=\tan^{-1}\left(\frac{4\sqrt{x}}{1-4x}\right)$

Q15. Find $\frac{dy}{dx}$ of $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Q16. If $siny = xsin(a+y)$, then prove that $\frac{dy}{dx}=\frac{sin^2(a+y)}{sina}$

Q17. If $y = \sqrt{tanx + \sqrt{tanx + \sqrt{tanx \ldots \ldots \infty}}}$ prove that $\frac{dy}{dx} = \frac{sec^2x}{2y-1}$

Q18. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ *then* $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

Q19. Differentiate $cosx^{cosx}$ *wrt* x

Q20. $y = x^{sinx}$, then find $\frac{dy}{dx} = x^{sinx}\left(\frac{\sin x}{x} + \cos x \log x\right)$

Q21. $x = cos\theta + cos2\theta$ *and* $y = sin\theta + sin2\theta$, then find $\frac{dy}{dx} = \frac{cos\theta + 2cos2\theta}{-(sin\theta + 2sin2\theta)}$

Q22. Diff. x^2 w.r.t x^3

Application of derivative

Q1. Find the second derivative of functions: $\log \log x$

Q2. If $y = x^x$ show that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

Q3. If $y = e^{ax} \sin bx$, show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$

Q4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point where x=3

Q5. Find the slope of the normal to the curve $x = acos^3\theta, y = asin^3$ at $\theta = \frac{\pi}{4}$

Q6. Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) are at right angles.

Q7. Find the equations of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

Q8. Find the radius of curvature of the curve $y = \log \sin x$ at $x = \frac{\pi}{2}$

Q9. Find the radius of curvature for the curve $x = acos^3\theta, y = asin^3$ at $\theta = \frac{\pi}{4}$

Q10. Find the maximum and minimum value of

$$f(x) = x^3 - 12x^2 + 36x + 17 \text{in } 1 \leq x \leq 10$$

Q11. Determine the max. and min. values of the following function in the stated domain

$$y = 2cos2x - cos4x, 0 \leq x \leq \pi$$

Q12. Find the local max values of $(x-1)(x+2)^2$

Co-ordinate Geometry

Q1. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2,3).

Q2. Find the equation of a straight line on which length of perpendicular from the origin is 4 units and the line makes an angle of 120^0 with the positive direction of x – axis.

Q3. By using the concept of equation of a line, prove that the three points (3,0), (-2, -2) and (8,2) are collinear.

Q4. Find the equation of line passing through (1,2) and make angle 30^0.

Q5. Point R(h,k) divides a line segment between the axes in the ratio 1:2. Find the equation of line.

Q6. The perpendicular from the origin to a line meets it at the point (-2,9). Find the equation of a line

Q7. Find the coordinates of the foot of the perpendicular from the point (2,3) on the line

$x + y - 11 = 0$.

Q8. Find the length and the equation of the perpendicular from (3,4) to the line

$3x - 4y + 1 = 0$

Q9. What are the points on the y-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Q10. Find the coordinates of a point on $x + y + 3 = 0$ whose perpendicular distance from $x + 2y + 2 = 0$ is $\sqrt{5}$.

Q11. Find the equation of the circle which is concentric with the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ and which passes through the point (5,-2).

Q12. Find the equation of the circle which touches x-axis and whose centre is (1,2).

Q13. Find the equation of the circle which passes through the three points (1,0), (0,1) and (1, -2).

Q14. Find the condition that the straight line $lx + my + n = 0$ may touch the parabola

$y^2 = 4ax$

Q15. If the line $2x + 3y = 1$ touches the parabola $y^2 = 4ax$ then find the length of the latus rectum.

Q16. Find the equation of ellipse whose focus is (3,4) the corresponding directrix is $3x + 4y - 5 = 0$ and the eccentricity is $\frac{2}{3}$.

Q17. Find the condition that the line $lx + my + n = 0$ may touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Q18. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 15$ which are parallel to $y = 2x + k$

Q19. Taking the axes of coordinates as the axes, find the equation of the equation of the hyperbola whose focus is (2,2) the directrix $x + y = 9$ and eccentricity is 2.

Q20. Find the eccentricity and the co-ordinates of the foci of the hyperbola $3x^2 - y^2 = 4$.

Probability

Q1. Three dice are thrown simultaneously. Find the probability of getting a total of at least 6.

Q2. Find the probability that a leap year, selected at random will contain 53 Sundays.

Q3. A card is drawn from an ordinary pack and a gambler bet that it is a spade or an ace. What are the odds against his winning the bets?

Q4. A committee of 5 persons is to be constituted from a group of 6 male and 8 female. If the selection is made randomly, find the probability that there are three female and 2 males in the committee.

Q5. The letters of the word 'SOCIETY' are placed at random in arow. What is the probability that three vowels come together?

Q6. A five-digit numbers is formed from the digits 1,2,3,4,5,7 without repetition. Find the probability that the number is divisible by 4.

Q7. Given $P(E) = \frac{3}{5}$ and $P(E) = \frac{1}{5}$ find $P(E \text{ or } F)$, if E and F are mutually exclusive events.

Q8. Given that the events E and F are such that $P(E) = \frac{1}{2}$ and $P(EUF) = \frac{3}{5}$ *and* $P(F) = P$. Find P, if E and F are mutually exclusive events.

Q9. A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is a king or a heart or a red card.

Q10. Two coins are tossed. What is the probability of coming up of two heads, if it is known that at least on head comes up?

Q11. Events E and F are independent. Find $P(F)$, if $P(E) = \frac{2}{5}$ and $P(EUF) = \frac{3}{5}$.

Q12. Events E and F are such that $P(E) = \frac{1}{2}$, $P(F) = \frac{7}{12}$ and $P(not\ E\ or\ not\ F) = \frac{1}{4}$. State whether E and F are independent.

Q13. Find the mean for the following data:

AGE	NO. OF PERSON
Less than 20	16
Less than 30	21
Less than 40	35
Less than 50	52
Less than 60	58
Less than 70	78
Less than 80	94
Less than 90	100

Q14. Calculate the median marks of the students for the following data:

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of student	8	10	22	40	20

Q15. Weight of the 40 eggs were recorded as given below:

Weight (gram)	85-89	90-94	95-99	100-104	105-109
No. of Eggs	10	12	12	4	2

Calculate the median weight, correct up to 2 decimal places.

Q16. Find the mode of the following distribution:

Marks obtained	10-24	25-39	40-54	55-69	70-84	85-99
No. of student	25	29	23	19	14	10

Q17. Find the Mean, Variance and Standard Deviation for the following set of numbers:
25,50,45,70,42,36,48,34,60

Q18. Find the Mean, Variance and Standard Deviation for the following data:

X_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Q19. Calculate the standard deviation for the following age distribution of persons:

Age (in Years)	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Persons	3	61	132	153	140	51	2

Q20. The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2,4,10,12 and 14. Find the remaining two observations.